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Module: University Physics 2 (BDIC2008J)
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Problem Set 5

Magnetic Fields

Problem 1. *An alpha particle travels at a velocity $\vec{v}$ of magnitude* $550\,\mathrm{m/s}$ *through a uniform magnetic field $\vec{B}$ of magnitude 0.045* T. *(An alpha particle has a charge of* $+3.2\times10^{-19}\,\mathrm{C}$ *and a mass of* $6.6\times10^{-27}\,\mathrm{kg}$*.) The angle between $\vec{v}$ and $\vec{B}$ is 52°. What is the magnitude of **(a)** the force $\vec{F}_B$ acting on the particle due to the field and **(b)** the acceleration of the particle due to $\vec{F}_B$? **(c)** Does the speed of the particle increase, decrease, or remain the same?*

Solution. **(a)** We use the Lorentz Equation:

$$F_B = |q|vB\sin\phi = (+3.2\times10^{-19}\,\mathrm{C})(550\,\mathrm{m/s})(0.045\,\mathrm{T})(\sin 52^\circ) = 6.2\times10^{-18}\,\mathrm{C}$$

(b) The acceleration is

$$a = F_B/m = (6.2\times10^{-18}\,\mathrm{C})/(6.6\times10^{-27}\,\mathrm{kg}) = 9.5\times10^{8}\,\mathrm{m/s^2}$$

(c) Since it is perpendicular to $\vec{v}$, $\vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem, both the kinetic energy and the speed of the particle remain unchanged. □

Problem 2. *A proton travels through uniform magnetic and electric fields. The magnetic field is* $B = -2.50\hat{\imath}\,\mathrm{mT}$*. At one instant, the velocity of the proton is* $\vec{v} = 2000\hat{\jmath}\,\mathrm{m/s}$*. At that instant and in unit-vector notation, what is the net force acting on the proton if the electric field is* ***(a)*** $4.00\hat{k}\,\mathrm{V/m}$*,* ***(b)*** $-4.00\hat{k}\,\mathrm{V/m}$*, and* ***(c)*** $4.00\hat{\imath}\,\mathrm{V/m}$*? (Hint: The type* $3\,\mathrm{N}\hat{\imath} + 4\,\mathrm{N}\hat{\jmath} + 5\,\mathrm{N}\hat{k}$ *is called unit-vector notation.)*

Solution. **(a)** The net force on the proton is given by

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.60\times10^{-19}\,\mathrm{C})[(4.00\,\mathrm{V/m})\hat{k} + (2000\,\mathrm{m/s})\hat{\jmath}\times(-2.50\times10^{-3}\,\mathrm{T})\hat{\imath}]\\ &= (1.44\times10^{-18}\,\mathrm{N})\hat{k}\end{aligned}$$

(b) In this case, we have

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.60\times10^{-19}\,\mathrm{C})[(-4.00\,\mathrm{V/m})\hat{k} + (2000\,\mathrm{m/s})\hat{\jmath}\times(-2.50\times10^{-3}\,\mathrm{T})\hat{\imath}]\\ &= (1.60\times10^{-19}\,\mathrm{N})\hat{k}\end{aligned}$$

(c) In the final case, we have

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.60\times10^{-19}\,\mathrm{C})[(4.00\,\mathrm{V/m})\hat{\imath} + (2000\,\mathrm{m/s})\hat{\jmath}\times(-2.50\times10^{-3}\,\mathrm{T})\hat{\imath}]\\ &= (6.41\times10^{-19}\,\mathrm{N})\hat{\imath} + (8.01\times10^{-19}\,\mathrm{N})\hat{k}\end{aligned}$$

□

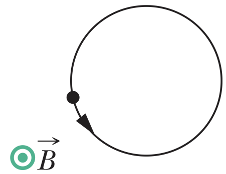
Problem 3. *An ion source is producing* ^{6}Li *ions, which have charge* $+e$ *and mass* $9.99 \times 10^{-27}\,\mathrm{kg}$*. The ions are accelerated by a potential difference of* $10\,\mathrm{kV}$ *and pass horizontally into a region in which there is a uniform vertical magnetic field of magnitude* $B = 1.2\,\mathrm{T}$*. Calculate the strength of the smallest electric field, to be set up over the same region, that will allow the* ^{6}Li *ions to pass through undeflected.*

Solution. Since the total force given by $\vec{F} = e|\vec{E} + \vec{v} \times \vec{B}|$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $mv^2/2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = (1.2\,\mathrm{T})\sqrt{\frac{2(1.60 \times 10^{-19}\,\mathrm{C})(10 \times 10^3\,\mathrm{V})}{(9.99 \times 10^{-27}\,\mathrm{kg})}} = 6.8 \times 10^5\,\mathrm{V/m}$$

□

Problem 4. *In the Figure below, a particle moves along a circle in a region of uniform magnetic field of magnitude* $B = 4.00\,\mathrm{mT}$*. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude* $3.20 \times 10^{-15}\,\mathrm{N}$*. What are* ***(a)*** *the particle's speed,* ***(b)*** *the radius of the circle, and* ***(c)*** *the period of the motion?*



Solution. With the $\vec{B}$ pointing "out of the page", we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent toward the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Lorentz Equation (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e|\vec{B}|} = 4.99 \times 10^6\,\mathrm{m/s}$$

(b) Using $r = \dfrac{mv}{|q|B}$, we find $r = 0.0071\,\mathrm{m}$.

(c) Using $T = \dfrac{2\pi m}{|q|B}$, we find $T = 8.93 \times 10^{-9}\,\mathrm{s}$. □

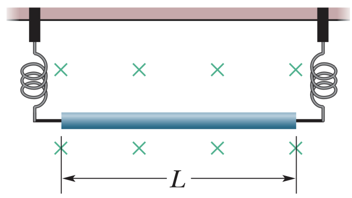
Problem 5. *An electron follows a helical path in a uniform magnetic field of magnitude* $0.300\,\mathrm{T}$*. The pitch of the path is* $6.00\,\mu\mathrm{m}$*, and the magnitude of the magnetic force on the electron is* $2.00 \times 10^{-15}\,\mathrm{N}$*. What is the electron's speed?*

Solution. The distance traveled parallel to $\vec{B}$ is $d_\parallel = v_\parallel T = v_\parallel(2\pi m_e/|q|B)$. Thus,

$$v_\parallel = \frac{d_\parallel e B}{2\pi m_e} = 50.3\,\mathrm{km/s}$$

using the values given in this problem. Also, since the magnetic force is $|q|Bv_\perp$, then we find $v_\perp = 41.7\,\mathrm{km/s}$. The speed is therefore $v = \sqrt{v_\perp^2 + v_\parallel^2} = 65.3\,\mathrm{km/s}$. $\square$

Problem 6. *A* 13.0 g *wire of length* $L = 62.0\,\mathrm{cm}$ *is suspended by a pair of flexible leads in a uniform magnetic field of magnitude* 0.440 T *(see the Figure below). What are the* ***(a)*** *magnitude and* ***(b)*** *direction (left or right) of the current required to remove the tension in the supporting leads?*



Solution. **(a)** The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other, the magnitude of the magnetic force is given by $F_B = ILB$, where L is the length of the wire. Thus,

$$ILB = mg \Rightarrow I = \frac{mg}{LB} = \frac{(0.0130\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(0.620\,\mathrm{m})(0.440\,\mathrm{T})} = 0.467\,\mathrm{A}$$

(b) Applying the right-hand rule reveals that the current must be from left to right. □

Problem 7. *A single-turn current loop, carrying a current of* $4.00\,\mathrm{A}$*, is in the shape of a right triangle with sides* 50.0*,* 120*, and* $130\,\mathrm{cm}$*. The loop is in a uniform magnetic field of magnitude* $75.0\,\mathrm{mT}$ *whose direction is parallel to the current in the* $130\,\mathrm{cm}$ *side of the loop. What is the magnitude of the magnetic force on* ***(a)*** *the* $130\,\mathrm{cm}$ *side,* ***(b)*** *the* $50\,\mathrm{cm}$ *side, and* ***(c)*** *the* $120\,\mathrm{cm}$ *side?* ***(d)*** *What is the magnitude of the net force on the loop?*

Solution. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $l_y = 50\,\mathrm{cm}$ side runs along the $+y$ axis, while the $l_x = 120\,\mathrm{cm}$ side runs along the $+x$ axis. The angle made by the hypotenuse (of length $130\,\mathrm{cm}$) is

$$\theta = \arctan(50/120) = 22.6\,^\circ$$

relative to the $120\,\mathrm{cm}$ side. If one measures the angle counterclockwise from the $+x$ direction, then the angle for the hypotenuse is $180\,^\circ - 22.6\,^\circ = +157\,^\circ$. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the $+z$ axis. We take $\vec{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750\,\mathrm{T}$,

$$B_x = -B\cos\theta = -0.0692\,\mathrm{T}, \qquad B_y = B\sin\theta = 0.0288\,\mathrm{T}$$

(a) Ampere's Force $F = IL \times B$ produces zero force when $\vec{L} \parallel \vec{B}$ so there is no force exerted on the hypotenuse of length $130\,\mathrm{cm}$.

(b) On the $50\,\mathrm{cm}$ side, the B_x component produces a force $il_yB_x\hat{k}$, and there is no contribution from the B_y component. Using SI units, the magnitude of the force on the l_y side is therefore

$$(4.00\,\mathrm{A})(0.500\,\mathrm{m})(0.0692\,\mathrm{T}) = 0.138\,\mathrm{N}$$

(c) On the $120\,\mathrm{cm}$ side, the B_y component produces a force $il_xB_y\hat{k}$, and there is no contribution from the B_x component. Using SI units, the magnitude of the force on the l_x side is therefore

$$(4.00\,\mathrm{A})(1.20\,\mathrm{m})(0.0288\,\mathrm{T}) = 0.138\,\mathrm{N}$$

(d) The net force is

$$il_yB_x\hat{k} + il_xB_y\hat{k} = 0,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\vec{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$, but $B_y < 0$ and a zero net force would still be the result. $\square$

Problem 8. *A wire* $50.0\,\text{cm}$ *long carries a* $0.500\,\text{A}$ *current in the positive direction of an* x*-axis through a magnetic field* $\vec{B} = (3.00\,\text{mT})\hat{j} + (10.0\,\text{mT})\hat{k}$*. In unit-vector notation, what is the magnetic force on the wire?*

Solution. The magnetic force on the wire is

$$\begin{aligned}\vec{F}_B &= I\vec{L} \times \vec{B} = IL\hat{i} \times (B_y\hat{j} + B_z\hat{k}) = IL(-B_z\hat{j} + B_y\hat{k}) \\ &= (0.500\,\text{A})(0.500\,\text{m})[-(0.01\,\text{T})\hat{j} + (0.003\,\text{T})\hat{k}] \\ &= (-2.50 \times 10^{-3}\hat{j} + 0.750 \times 10^{-3}\hat{k})\,\text{N}\end{aligned}$$

□

Problem 9. *A circular wire loop of radius* $15.0\,\mathrm{cm}$ *carries a current of* $2.60\,\mathrm{A}$. *It is placed so that the normal to its plane makes an angle of* 41.0° *with a uniform magnetic field of magnitude* $12.0\,\mathrm{T}$. ***(a)*** *Calculate the magnitude of the magnetic dipole moment of the loop.* ***(b)*** *What is the magnitude of the torque acting on the loop?*

Solution. **(a)** $\mu = NAI = \pi r^2 I = \pi(0.150\,\mathrm{m})^2(2.60\,\mathrm{A}) = 0.184\,\mathrm{A}\cdot\mathrm{m}^2$

(b) The torque is

$$\tau = |\vec{\mu}\times\vec{B}| = \mu B\sin\theta = (0.184\,\mathrm{A}\cdot\mathrm{m}^2)(12.0\,\mathrm{T})\sin 41.0^\circ = 1.45\,\mathrm{N}\cdot\mathrm{m}$$

□

Problem 10. *The magnetic dipole moment of Earth has magnitude* 8.00×10^{22} J/T. *Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is* 3500 km, *calculate the current they produce.*

Solution. From $\mu = NIA = I\pi r^2$ we get

$$I = \frac{\mu}{\pi r^2} = \frac{8.00 \times 10^{22}\,\mathrm{J/T}}{\pi(3500 \times 10^3\,\mathrm{m})^2} = 2.08 \times 10^9\,\mathrm{A}$$

□